

Interpretation according to Bohr, von Neumann, and Dirac

Lecturer: Joseph Emerson

Overview of the Significant Features of Quantum Theory

Wave-particle duality: coherent superpositions and interference effects due to wave-like properties for particles (e.g. two-slit experiment) and the particle-like behaviour of EM waves (e.g., photo-electric effect).

What is the physical meaning of a coherent superposition of distinct position eigenstates of an electron?

Invasive role of measurement: the necessity of state disturbance and the intrinsic limitations of the measurement process (e.g., Heisenberg microscope).

Indeterminism and uncertainty: Robertson's uncertainty principle (the unavoidable dispersion for non-commuting observables) and the loss of determinism (causality).

There are two possible explanations for the characteristic dispersions that are predicted by quantum theory: i) a fundamental randomness (stochasticity) in nature, or ii) the existence of additional (hidden) coordinates which fully determine the experimental outcomes. Next week we take this up by considering the EPR argument for incompleteness of the wavefunction, and then turn to a discussion of the (known) constraints on hidden variables (non-locality and contextuality).

The ambiguous ontology of quantum states: The quantum measurement problem for the orthodox interpretation arises from the assumption that quantum states provide a complete description of physical properties. A related problem is to understand the physical status of the projection postulate and an unambiguous set of rules for defining when it may be correctly applied.

The Measurement Process:

Projection postulate: Upon measurement of a non-degenerate observable $R = \sum \lambda_k |\phi_k\rangle\langle\phi_k|$, the following transformation is required:

$$\rho(t) \rightarrow \rho'(t) = \sum_k \langle\phi_k|\rho(t)|\phi_k\rangle |\phi_k\rangle\langle\phi_k| \quad (1)$$

This is von Neumann's process 1, also called the 'collapse postulate', or 'state reduction.'

Projection destroys the coherence of the state (across the eigenstates of R) and eliminates the possibility of observing interference effects from the prior phase-relations in subsequent experiments.

This entropy-increasing rule (representing a pure $\rightarrow$ mixed transition) applies when the actual outcome is 'ignored.'

If we wish to describe the state of a sub-ensemble which is post-selected based on a certain outcome (e.g., the outcome $\{\lambda_j\}$), then the quantum state operator for that sub-ensemble is,

$$\rho(t) \rightarrow \rho'(t) = |\phi_j\rangle\langle\phi_j|. \quad (2)$$

In particular, this rule applies *when describing an individual system* where outcome λ_j has been obtained.

The post-selected projection postulate can not be deduced from the unitary transformation law (von Neumann's 'process 2.') on either the system alone, or on the system combined with some other auxiliary systems.

In the most general measurement of an observable, the system is not left in an eigenstate. For example, when a photon's momentum is measured by absorption it is clear that the photon is not left in a momentum eigenstate - it is destroyed by the measurement. Pauli calls the ideal von Neumann measurements "measurements of the first kind." Measurements of the first kind, combined with post-selection, are an important means of "state preparation."

von Neumann and Dirac on the Measurement Process:

The projection postulate can not be deduced from the unitary dynamics:

“We have then answered the question as to what happens in the measurement of [an observable]. To be sure, the “how” remains unexplained for the present. This discontinuous transition from the wavefunction into one of [the eigenstates of the observable] is certainly not of the type described by the time dependent Schrödinger equation. This latter always results in a continuous change of [the wavefunction], in which the final result is uniquely determined and is dependent on [the wavefunction].”

von Neumann (1932/1955), p. 217

The projection postulate is demanded for consistency with experiment:

“From physical continuity, if we make a second measurement of the same dynamical variable immediately after the first, the result of the second measurement must be the same as that of the first.”

Dirac (1947)

von Neumann goes through a long analysis to show that the application of the projection postulate can be applied in a consistent way either to the system directly or to the system + apparatus, but insists that ultimately the postulate must be applied to describe the outcomes obtained by the act of observation:

“That is, we must always divide the world into two parts, the one being the observed system, the other the observer. In the former, we can follow up all physical processes (in principle at least) arbitrarily precisely. In the latter, this is meaningless. The boundary between the two is arbitrary to a very large extent. ... That this boundary can be pushed arbitrarily deeply into the interior of the body of the actual observer is the content of the principle of the psycho-physical parallelism - but this does not change the fact that in each method of description the boundary must be put somewhere, if the method is not to proceed vacuously, i.e., if a comparison with experiment is to be possible. Indeed experience only makes statements of this type: an observer has made a certain (subjective) observation; and never any like this: a physical quantity has a certain value.

Now quantum mechanics describes the events which occur in the observed portions of the world, so long as they do not interact with the observing portion, with the aid of the process 2 [Schrodinger evolution], but as soon as such an interaction occurs, i.e., a measurement, it requires the application of the process 1 [projection postulate]. The dual form is therefore justified.”

von Neumann (1932/1955), p. 418-419

What does it mean to interpret a theory?

The minimum interpretation that is required is to develop the most general set of rules that enable the theory to predict experimental outcomes. These operational bridge principles serve a purely epistemological role (from the Greek *episteme*, meaning ‘knowledge’).

Operationalism:

“In a strict sense, quantum theory is a set of rules allowing computation of probabilities for the outcomes of tests which follow specified preparations.”

A. Peres (1995), p 13.

The idea of operationalism is to relate the elements of the mathematical formalism to the operations one can perform in the lab and to the outcomes one can observe. This level of interpretation fulfills the practical role of a physical theory as a means of predicting and controlling physical systems.

Operational interpretations are the minimum needed for the ‘shut up and calculate’ application of quantum theory.

“Ordinary quantum mechanics (as far as I know) is just fine FOR ALL PRACTICAL PURPOSES [FAPP]”

It should be emphasized from this operational context that quantum theory can make at most only statistical predictions, i.e., statistics with finite dispersion for some observables.

But what is really going on?

In addition to the practical solution offered by operationalism, it is interesting and important to ask: what can we say about the underlying reality that is consistent with quantum theory. That is, we demand a set of *ontological* bridge principles (from the Greek *ontos*, meaning ‘to be’). These principles, or correspondence rules, are needed also to develop the explanatory aspect of a physical theory, that is, to provide some explanation or story about the nature of the physical world.

The ontological bridge principles can also be practical in the sense that they can give intuition about how systems behave and about what results to expect when direct calculation is infeasible. Moreover, the answers can hopefully give insight into how to further develop and unify our physical theories (e.g., the problem of unifying quantum theory with gravity.)

Ontological question for quantum theory:

How do the mathematical elements of quantum theory correspond to physical properties of nature?

What is the physical status of quantum states, e.g, are they complete?

What constraints does quantum theory impose on the possibility of assigning objective properties to observables?

Is a fully deterministic description in terms of objective properties possible?

Interpretation of classical physics: ontology vs epistemology

A good example of an ontological theory is Hamiltonian mechanics: definite objective properties (q, p) may be assigned to a system and these properties evolve deterministically according to the canonical equations:

The state $(q(t), p(t))$ is complete in the sense that these values (combined with the dynamical laws) provide all the information that is needed to determine the physical state of the system at any time.

A good example of an epistemological classical theory is Liouville mechanics. Here we have a state $\rho(q(t), p(t), t)$ which is a probability density that evolves deterministically according to:

The state $\rho(q, p, t)$ represents (incomplete, subjective) knowledge about the system's physical properties.

There is a close correspondence between the Liouville state ρ and the quantum state ρ and Liouville mechanics and quantum mechanics.

1. The deterministic dynamical law is similar to the Schrodinger evolution. In fact, due to Koopman's theorem, the Liouville time-evolution may be represented by a unitary operator (sometimes called the Frobenius-Perron operator).
2. The outcomes upon measurement are (subjectively) random.
3. An independent transformation rule is required to account for the state after measurement: a classical collapse postulate is required to describe the state after measurement (this transformation is non-unitary).

The Copenhagen Interpretation:

Defining complementarity:

“Complementarity: any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena.”

Bohr (1934), p. 10

Causality and maybe also objective physical reality must be rejected:

“... a subsequent measurement to a certain degree deprives the information given by a previous measurement of its significance for predicting the future course of the phenomena. Obviously, these facts not only set a limit to the extent of the information obtainable by measurements, but they also set a limit to the meaning which we may attribute to such information. We meet here in a new light the old truth that in our description of nature the purpose is not to disclose the real essence of the phenomena but only to track down, so far as it is possible, relations between the manifold aspects of our experience.”

Bohr (1934), p. 18

“An independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation.”

Bohr (1928)

There is a necessary boundary between quantum and classical:

Bohr holds firm to the fact that although “atomic phenomena” must be described by quantum mechanics, our measuring devices must be described using classical physics:

“The experimental conditions can be varied in many ways but the point is that in each case we must be able to communicate to others what we have done and what we have learned, and that therefore the functioning of the measuring instruments must be described within the framework of classical physical ideas.”

Bohr (1934), p. 89

But by what criterion can we determine where the boundary arises? Bohr concedes there is none:

There is “the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear.”

Bohr (1949), p. 10

Were Bohr’s views informed by quantum mechanics, or imposed upon quantum mechanics?

“How closely the idea of complementarity was in accord with Bohr’s older philosophical ideas became apparent through an episode which took place ... on a sailing trip from Copenhagen to Svendborg on the Island Fyn... Bohr was full of the new interpretation of quantum theory, and as the boat took us full sail southward ... there was plenty of time to reflect philosophically on the nature of atomic theory. ... Finally, one of Bohr’s friends remarked drily, ‘But Niels, this is not really new, you said exactly the same ten years ago.’ ”

Heisenberg (1967)

The ‘Orthodox’ Dirac-von Neumann Interpretation:

The quantum state gives a complete description of physical properties:

“In this method of description, it is evident that everything which can be said about the state of a system must be derived from its wave function.”

von Neumann (1932/1955), p. 196

The very limited conditions under which observables have actual, definite values are prescribed by the ‘eigenvalue-eigenstate link’ (term due to A. Fine, 1973):

“The expression that an observable ‘has a particular value’ for a particular state is permissible in quantum mechanics in the special case when a measurement of the observable is certain to lead to the particular value, so that the state is an eigenstate of the observable. In the general case we cannot speak of an observable having an value for a particular state, but we can speak of its having an average value for the state. We can go further and speak of the probability of its having any specified value for the state, meaning the probability of this specified value being obtained when one makes a measurement of the observable.”

Dirac (1958)

The projection postulate introduces fundamental randomness and we must reject ‘the principle of sufficient cause’:

“This concept of quantum mechanics, which accepts its statistical expression as the actual for of the laws of nature, and which abandons the principle of causality, is the so-called statistical interpretation.”

von Neumann (1932/1955), p. 210

The projection postulate represents a physical transformation or ‘jump’:

“When we measure a real dynamical variable, the disturbance involved in the act of measurement causes a jump in the state of the dynamical system. From physical continuity, if we make second measurement of the same dynamical variable immediately after the first, the result of the second measurement must be the same as that of the first. Thus after the first measurement has been made, there is no indeterminacy in the result of the second. Hence, after the first measurement has been made, the system is in an eigenstate of the dynamical variable, the eigenvalue it belongs to being equal to the result of the first measurement. This conclusion must still hold if the second measurement is not actually made. In this way we see that a measurement always causes the system to jump into an eigenstate of the dynamical variable that is being measured, the eigenvalue this eigenstate belongs to being equal to the result of the measurement.”

Dirac (1958)

The concept of determinism must be abandoned:

“The question of causality could be put to a true test only in the atom, in the elementary processes themselves, and here everything in the present state of our knowledge militates against it. The only formal theory existing at the present time which orders and summarizes our experiences in this area in a half-way satisfactory manner, i.e., quantum mechanics, is in compelling logical contradiction with causality. Of course it would be an exaggeration to maintain that causality has thereby been done away with: quantum mechanics has, in its present form, several serious lacunae, and it may even be that it is false, although this latter possibility is highly unlikely, in the face of its startling capacity in the qualitative explanation of general problems, and in the quantitative calculation of special ones.”

von Neumann (1932/1955), p. 327

Dissent from Einstein

On the Copenhagen Interpretation:

“The Heisenberg-Bohr tranquilizing philosophy - or religion? - is so delicately contrived that, for time being, it provides a gentle pillow for the true believer from which he cannot very easily be aroused. So let him lie there.”

A. Einstein (1928)

Against the orthodox view that quantum mechanics provides a complete description of individual systems:

“One arrives at very implausible theoretical conceptions, if one attempts to maintain the thesis that the statistical quantum theory is in principle capable of producing a complete description of an individual physical system I am convinced that everyone who will take the trouble to carry through such reflections conscientiously will find himself finally driven to this interpretation of quantum-theoretical description (the ψ -function is to be understood as the description not of a single system but of an ensemble of systems) . . . There exists, however, a simple psychological reason for that fact that this most nearly obvious interpretation is being shunned. For if the statistical quantum theory does not pretend to describe the individual system (and its development in time) completely, it appears unavoidable to look elsewhere for a complete description of the individual system . . . Assuming the success of efforts to accomplish complete physical description, the statistical quantum theory would, within the framework of future physics, take an approximately analogous position to the statistical mechanics with the framework of classical mechanics. I am rather firmly convinced that the development of theoretical physics will be of that type; but the path will be lengthy and difficult. ”

A. Einstein (1949)

The more complete analysis Einstein seeks “is *in principle* excluded.”

Bohr (1949), p. 35

In reference to the Copenhagen viewpoint:

“To believe this is logically possible without contradiction; but it is so very contrary to my scientific instinct that I cannot forego the search for a more complete description.”

Einstein (1936)

Have these issues been satisfactorily resolved ... ?

“ I think that I can safely say that nobody understands quantum mechanics”

R. Feynman (Nobel Laureate, 1965)

The measurement problem a la Schrodinger:

“A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny amount of radioactive substance, so small, that perhaps in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The first atomic decay would have poisoned it. The ψ -function of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared out in equal parts.

Schrodinger (1935)

The measurement problem, more formally:

Consider an atom described by a pure state corresponding to a coherent superposition of moving along two distinct trajectories. We arrange so that both trajectories pass through a detector such that a macroscopic pointer is moved to the 'left' if the atom is on the 'up' trajectory and to the 'right' if the atom is on the 'down' trajectory. If we treat the problem quantum mechanically and demand faithful measurements we get,

$$(\alpha|\text{up}\rangle + \beta|\text{down}\rangle) \otimes |\text{ready}\rangle \rightarrow \alpha|\text{up}\rangle \otimes |\text{left}\rangle + \beta|\text{down}\rangle \otimes |\text{right}\rangle, \quad (3)$$

such that after the interaction the apparatus pointer is in an entangled superposition with the atom's trajectory.

The orthodox view:

On the orthodox view, an observable property is definite only if the system is in an eigenstate of the operator associated with the observable and a pure state offers a complete description of the physical properties of a system. Our classical experience tells us that the position of the macroscopic pointer should be a definite property. However, the pointer is not in a position eigenstate.

Once we 'observe' the macroscopic pointer, we must apply the projection postulate, then the system + pointer state collapses to a product of position eigenstates, e.g. $|\text{down}\rangle \otimes |\text{right}\rangle$, and each has a definite position property. (It should be emphasized here that such a reduction of the state can not be derived from the unitary evolution of the system + whatever other degrees of freedom, and hence consists of an independent process.)

Criticism of the orthodox view:

How and when does the collapse process occur? In the case of Schrodinger's cat, is the cat's life status in an undefined state until it is observed? Or is the cat's own observation enough to collapse the wavefunction?

[Added after lecture] This point is perhaps emphasized best by Bell (in 'Against Measurement', 1990): "What exactly qualifies some physical systems to play the role of 'measurer'? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system . . . with a Ph D?"

This problem of what constitutes an observation has led some to speculate that there is an infinite regress until we ultimately consider our personal direct observations, that is, we may have to resign ourselves to solipsism. This is illustrated by the dilemma of "Wigner's friend" (Wigner, 1962), in which Wigner imagines a friend observing the outcome of an experiment, such as the pointer position, and points out the unitary evolution implies that his friend's mental state is in an entangled superposition with the system + apparatus state, and asks whether the collapse should occur only when Wigner himself asks his friend what he has observed.

The main problem is: which physical systems should we describe with quantum concepts and which should we describe with classical concepts? Here the old problem for Bohr of where to draw the boundary between quantum and classical descriptions is brought to a point against the orthodox interpretation.

The projection postulate has a natural analog in Liouville mechanics when a (finite-precision) measurement is performed. The required state update rule (after measurement) can not be described by the classical dynamical equation (e.g., it does not have a unitary representation). However, this is not conceptually problematic since the Liouville state is not presumed to be a physical (ontological) state with an objective status. Rather it has an epistemic status, and hence the transformation is not presumed to be a physical process. The classical projection corresponds to an update of the observer's information about the system.

For Einstein, the measurement problem is further evidence that the quantum description is not complete:

"The attempt to conceive the quantum-theoretical description as the complete description of the individual systems leads to unnatural theoretical interpretations, which become immediately unnecessary if one accepts the interpretation that the description refers to ensembles of systems and not to individual systems."

Einstein (1949)

Extra Quotes:

“...It is in principle impossible ...to formulate the basic concepts of quantum mechanics without using classical mechanics.”

Landau and Lifschitz, p. 2

“...Thus quantum mechanics occupies a very unusual place among physical theories: it contains classical mechanics as a limiting case, yet at the same time requires this limiting case for its own formulation.”

Landau and Lifschitz, p. 3

The debate with Einstein concerned “whether the renunciation of a causal mode of description of atomic processes ... should be regarded as a temporary departure from ideals to be ultimately revived or whether we are faced with an irrevocable step towards obtaining the proper harmony between analysis and synthesis of physical phenomena.”

Bohr (1949), p. 2

“Now from my conversations with Einstein I have seen that he takes exception to the assumption, essential to quantum mechanics, that state of a system is defined only by specification of an experimental arrangement. Einstein wants to know nothing of this. If one were able to measure with sufficient accuracy, this would of course be as true for small macroscopic spheres as for electrons. It is, of course, demonstrable by specifying though experiments, and I presume that you have mentioned and discussed some of these in your correspondence with Einstein, But Einstein has the philosophical prejudice that (for macroscopic bodies) a state (termed 'real') can be defined 'objectively' under any circumstances, that is, without specification of the experimental arrangement used to examine the system (of the macro-bodies), or to which the system is being 'subjected'. It seems to me that the discussion with Einstein can be reduced to this hypothesis of his, which I have called the idea (or the 'ideal') of the 'detached observer.' But to me and other representatives of quantum mechanics, it seems that there is sufficient experimental and theoretical evidence against the practicability of this ideal.”

W. Pauli - letter to ? (1954)

“First, it is inherently entirely correct that the measurement of the related process of the subjective perception is a new entity relative to the physical environment and is not reducible to the latter. Indeed, subjective perception leads us into the intellectual inner life of the individual, which is extra-observational by its very nature) since it must be taken for granted by any conceivable observation or experiment). Nevertheless, it is a fundamental requirement of the scientific viewpoint - the so-called principle of the psycho-physical parallelism - that it must be possible so to describe the extra-physical process of the subjective perception as if it were in reality in the physical world - i.e., to assign to its parts equivalent physical processes in the objective environment, in ordinary space. ”

von Neumann (1955), p. 418-419

“While the means of observation (experimental arrangements an apparatus, records such as spot on photographic plates) have still to be described in the usual ‘common language supplemented with the terminology of classical physics,’ the atomic ‘objects’ used in the theoretical interpretation of the ‘phenomena’ cannot any longer be described ‘in a unique way by conventional physical attributes.’ Those ‘ambiguous’ objects used in the description of nature have an obviously symbolic character.”

W. Pauli (1948)

“ the development of a state according to 1. is statistical, while according to 2. it is causal.”

von Neumann (1955), p. 356

“...one should expect that 2. would suffice to describe the intervention caused by a measurement. Indeed, a physical intervention can be nothing else than the temporary insertion of a certain energy coupling into the observed system, i.e., the introduction of an appropriate time dependency of H (prescribed by the observer). Why then do we need the special process 1. for the measurement?”

von Neumann (1955), p. 352